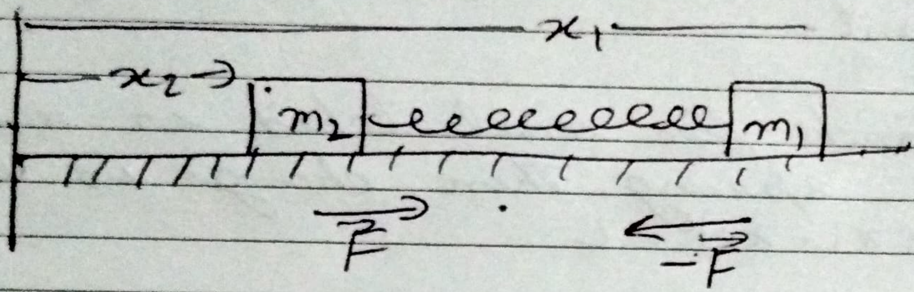
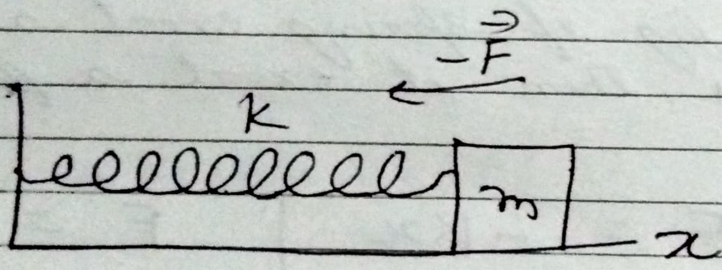
TWO BODY OSCILLATION

In a two body collision a spring connects two objects, each of which is free to move. When the objects are displaced and released they both oscillate.





Many examples of two body oscillations are found in nature like in diatomic molecules. We can visualize the molecule equivalent to two particles of masses m_1 and m_2 connected by a spring of spring constant k.

(a) Two oscillating bodies of masses m_1 and m_2 connected by a spring

(b) The relative motion can be represented by the oscellation of a single body having the reduced mass μ

The particle of masses m_1 and m_2 are located relative to origin O by the co-ordinates x_1 and x_2 respectively. The relative separation $(x_1 - x_2)$ gives the length of spring at any enstant.

If "L" stands for unstretched or relaxed length of spring, then change in length is $x = (x_1 - x_2) - L$

According to fig if spring exert a force "$-F$" on m_1, then it exert a force "$+F$" on m_2

$$F = -Kx$$

$$m_1 \frac{d^2 x_1}{dt^2} = -Kx$$

$$m_1 m_2 \frac{d^2 x_1}{dt^2} = -K m_2 x \quad \text{—— (a)}$$

$$F = Kx$$

$$m_2 \frac{d^2 x_2}{dt^2} = Kx$$

$$m_1 m_2 \frac{d^2 x_2}{dt^2} = K m_1 x \quad \text{—— (b)}$$

Subtract eq^n (b) from eq (a)

$$m_1 m_2 \frac{d^2 x_1}{dt^2} - m_1 m_2 \frac{d^2 x_2}{dt^2} = -K m_2 x - K m_1 x$$

$$m_1 m_2 \frac{d^2}{dt^2}(x_1 - x_2) = -(m_1 + m_2) K x$$

$$\frac{m_1 m_2}{(m_1 + m_2)} \frac{d^2}{dt^2}(x_1 - x_2) = -Kx$$

$$\mu \frac{d^2}{dt^2}(x_1 - x_2) = -Kx \quad \text{—— (1)}$$

Reduced mass

$$\mu = \frac{m_1 m_2}{(m_1 + m_2)}$$

$\because m_1 = m_2$

$$\mu = \frac{m}{2}$$

$m_1 \ll m_2 \quad \mu \approx m_1$

$$\therefore x = (x_1 - x_2) - L$$

$$\frac{dx}{dt} = \frac{d}{dt}\left[(x_1 - x_2) - L\right]$$

$$\Rightarrow v(t) = v_1(t) - v_2(t)$$

$$\therefore \frac{dL}{dt} = 0$$

$$\frac{d^2 x}{dt^2} = \frac{d^2}{dt^2}\left[(x_1 - x_2) - L\right]$$

$$\Rightarrow a(t) = a_1(t) - a_2(t)$$

using these values in eqn (1)

$$\therefore \mu \frac{d^2 x}{dt^2} = -Kx$$

$$\frac{d^2 x}{dt^2} = -\frac{K}{\mu} x$$

JULY 2012

M	T	W	T	F	S	S
30	31					1
2	3	4	5	6	7	8
9	10	11	12	13	14	15

AUGUST 2012

M	T	W	T	F	S	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad \text{———} \quad (ii)$$

9

10 This is identical relation as single oscillating mass

11

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

12

1

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

2 —— x ——